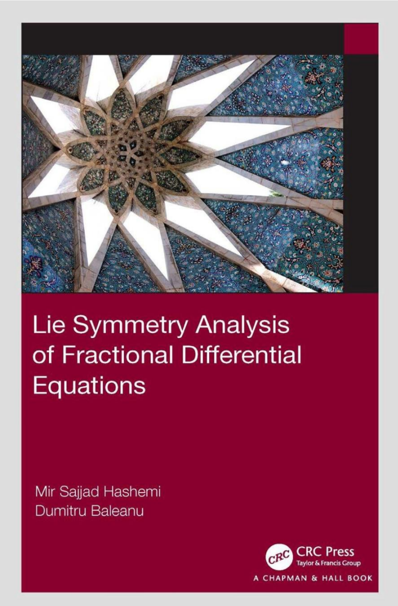

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Lie Symmetry Analysis of Fractional Differential Equations

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The Lie method (the terminology “the Lie symmetry analysis” and “the group analysis” are also used) is based on finding Lie’s symmetries of a given differential equation and using the symmetries obtained for the construction of exact solutions. The method was created by the prominent Norwegian mathematician Sophus Lie in the 1880s. It should be pointed out that Lie’s works on application Lie groups for solving PDEs were almost forgotten during the first half of the 20th century. In the end of the 1950s, L.V. Ovsiannikov, inspired by Birkhoff’s works devoted to application of Lie groups in hydrodynamics, rewrote Lie’s theory using modern mathematical language and published a monograph in 1962, which was the first book (after Lie’s works) devoted fully to this subject. The Lie method was essentially developed by L.V. Ovsiannikov, W.F. Ames, G. Bluman, W.I. Fushchych, N. Ibragimov, P. Olver, and other researchers in the 1960s–1980s. Several excellent textbooks devoted to the Lie method were published during the last 30 years; therefore, one may claim that it is the well-established theory at the present time. Notwithstanding the method still attracts the attention of many researchers and new results are published on a regular basis. In particular, solving the so-called problem of group classification (Lie symmetry classification) still remains a highly non-trivial task and such problems are not solved for several classes of PDEs arising in real world applications. Fractional calculus is an emerging field with ramifications and excellent applications in several fields of science and engineering. During the first attempt to think about what is derivative of order 1/2, stated by Leibniz in 1695, it was considered as a paradox as mentioned by L’Hopital. Since then the trajectory of the fractional calculus passed by several periods of intensive development both in pure and applied sciences. During the last few decades the fractional calculus has been associated with the power law effects and its various applications. It is a natural question to ask if the fractional calculus, as a non-local one, can produce new results within the well-established field of Lie symmetries and their applications. In fact, the fractional calculus was associated with the dissipative phenomena; therefore, it is a delicate question: can we have conservation laws for fractional differential equations associated to real world models? In our book we try to answer to this vital question by analyzing, mainly, some different aspects of fractional Lie symmetries and related conservation laws. Also, finding the exact solutions of a given fractional partial differential equation is not an easy task but we present this issue in our book. The book includes also a generalization of Lie symmetries for fractional integro-differential equations. Nonclassical Lie symmetries are discussed for fractional differential equations. Moreover, the invariant subspace method is considered to find the exact solutions of some fractional differential equations. In the present book, we assume the reader to be familiar with preliminaries of Lie symmetries for integer order differential equations. We believe that our book will be useful for PhD and postdoc graduates as well as for all mathematicians and applied researchers who use the powerful concept of Lie symmetries.